Overall strategy for Lagrange multipliers:

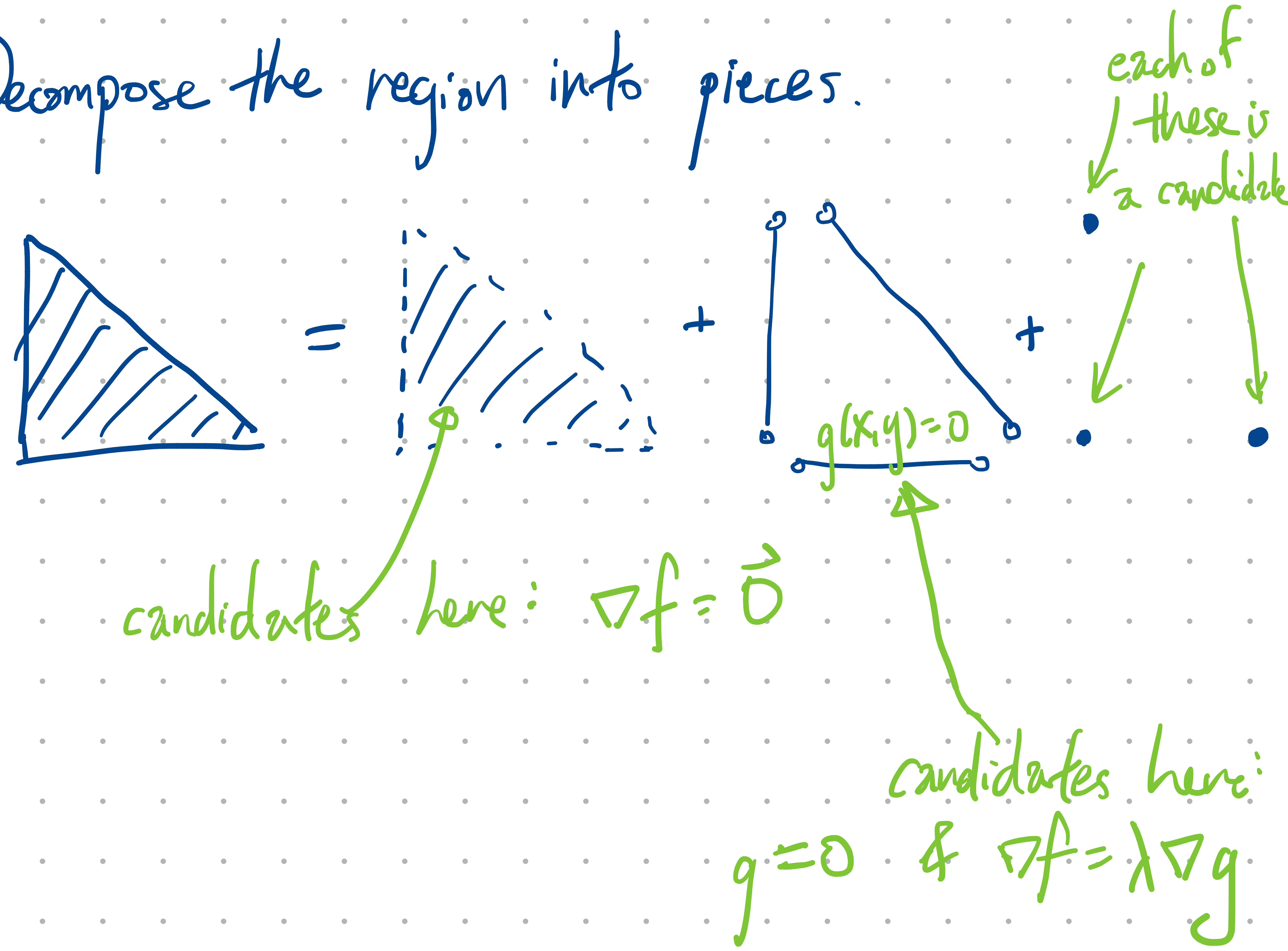
__Step 0:__ Use some argument to see that the extrema you're looking for __actually exist__.

e.g. if f is continuous, and the constraint region is closed and bounded, then can invoke EVT.

__Step 1:__ Identify potential candidates for extrema

- Decompose the region into pieces.

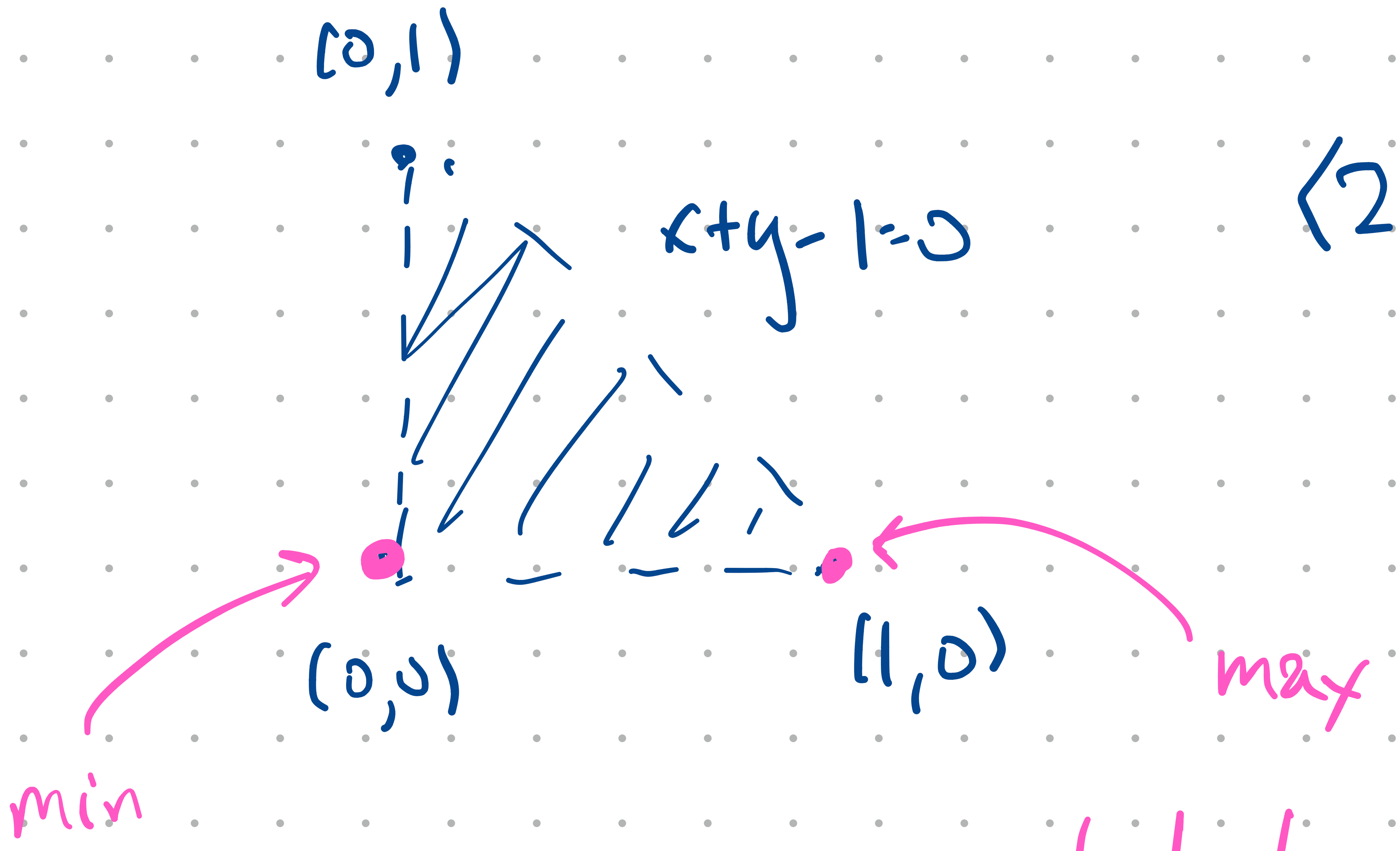
ex)



Step 2: Compare the values @ points identified in Step 1.

ex) $f(x,y) = 2x + y$

$\langle 2,1 \rangle = \langle 0,0 \rangle$ no sol.

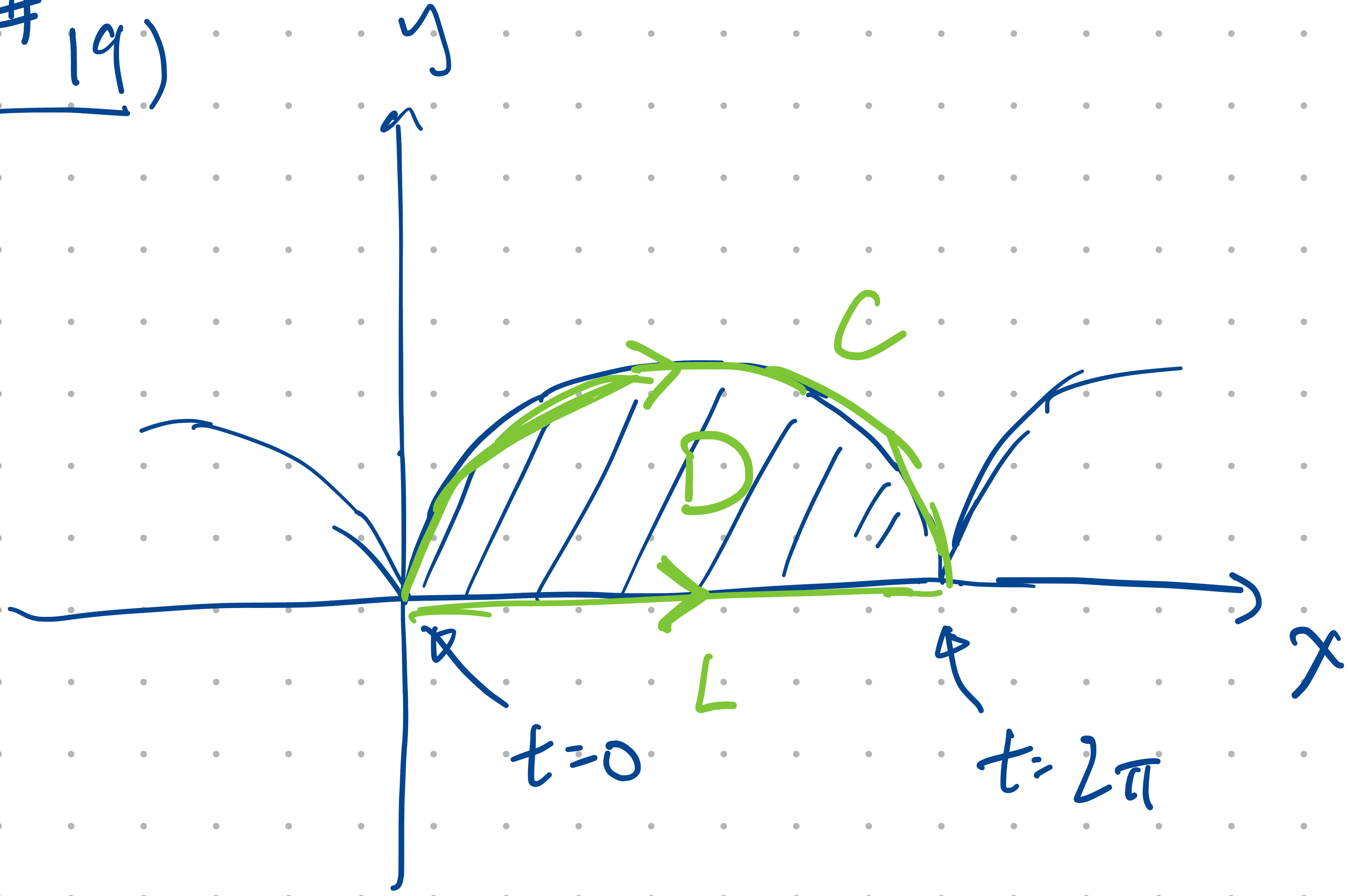


$\langle 2,1 \rangle = \lambda \langle 1,1 \rangle$

No solutions.

Need to check corners!

16.4 # 19)



$$\iint_D 1 \, dA = \oint_L \vec{F} \cdot d\vec{r} - \oint_C \vec{F} \cdot d\vec{r}$$

$\vec{F} = \langle P, Q \rangle$ is s.t. $Q_x - P_y = 1.$

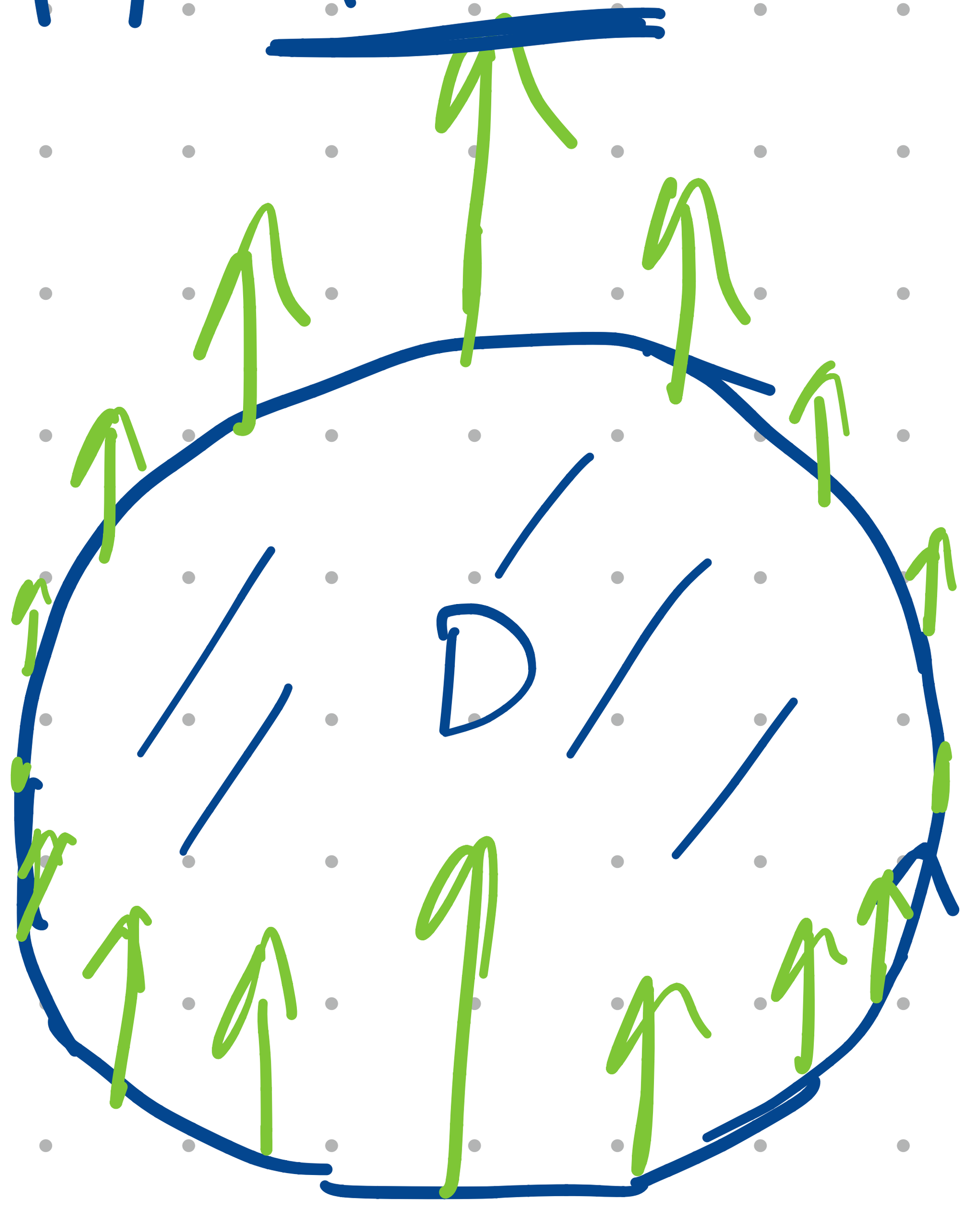
You choose $\vec{F}$. eg $\vec{F} = \langle 0, x \rangle$

or $\langle -y, 0 \rangle$

$d\vec{r} = \langle 1-\cos t, \sin t \rangle \, dt$ or $\langle -\frac{y}{2}, \frac{x}{2} \rangle$

etc...

#1) in 2D:



$\vec{F} = \langle 0, x^2 \rangle$

C: $x^2+y^2=1$, ccw.

Exercise) Check $\oint_C \vec{F}\cdot d\vec{r} = 0$.

But: $\vec{F}$ is not conservative

$Q_x - P_y = 2x$ is not the zero function on D

$\vec{F}$ is not always perp to C (see picture above)

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_D (Q_x - P_y)\,dA = 0$$

⚠ The three answer choices all imply $\oint_C \vec{F}\cdot d\vec{r} = 0$, but they do not follow from it.

#2) Just use Green's Thm:

$$\oint_C \langle 5+x^2, x\rangle \cdot d\vec{r} = \iint (1-0)\, dA = \text{Area of}$$

region inside C

#3)

b/c C is negatively oriented

$$\oint_C \boxed{x^3\, dy} = -\iint 3x^2\, dA = \text{negative.}$$

region enclosed by C

$$\langle 0, x^3\rangle \cdot d\vec{r}$$

$$\|$$

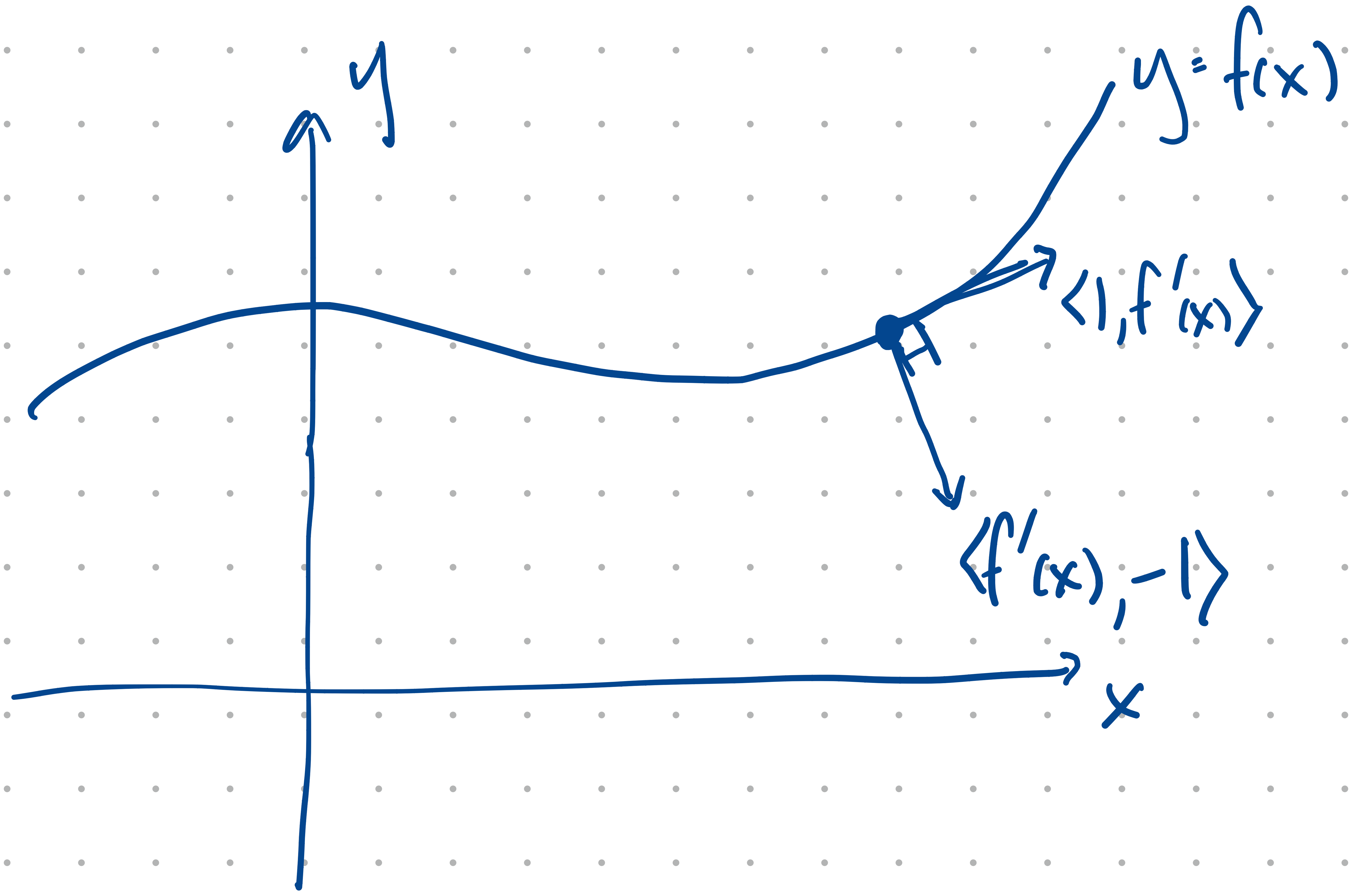
$$\langle dx, dy\rangle$$

#4) Interpret $z = f(x,y)$ as a <u>level set</u>:

$$\underbrace{f(x,y) - z}_{F(x,y,z)} = 0$$ is the "0-levelset" of F.

$$\nabla F(a,b,f(a,b)) = \langle f_x(a,b), f_y(a,b), -1 \rangle.$$

In SVC:



$f(x) - y = 0$

$\langle f'(x), -1 \rangle$

Please consult 212 Notes for remaining questions.